20th Australasian Fluid Mechanics Conference

Perth, Australia

5-8 December 2016

Local dissipation scales in turbulent jets

M. Xu[1], J. Zhang[2], J. Mi[2], A. Pollard[3]

[1] Marine Engineering College

Dalian Maritime University, Dalian, 116026, China

[2] State Key Laboratory of Turbulence & Complex Systems, College of Engineering

Peking University, Beijing 100871, China

[3] Department of Mechanical and Materials Engineering

Queen's University at Kingston, Ont., K7L 3N6, Canada

Abstract

This paper reports an experimental investigation of the characteristics of local dissipation length-scale field η in turbulent (round and square) jets with various jet-exit Reynolds numbers. Results reveal that the probability density function (PDF) of η, denoted by $Q(\eta)$, in the central fully-turbulent region, is insensitive to initial flow conditions and the departure from anisotropy. Excellent agreement is demonstrated with distributions previously measured from pipe flow and Direct Numerical Simulation (DNS) calculated from box turbulence. In the shear layer where the flow is not fully turbulent, $Q(\eta)$ exhibits higher probabilities at small η and the PDFs of velocity increments $\delta_L u$ across the integral length scale L are found to have exponential tails, suggesting the increased level of small-scale intermittency at these scales. This feature may come from the large-scale intermittency induced by the engulfment in the shear layer. In addition, the influence of the mean shear rate and Reynolds number on $Q(\eta)$ is negligible. Therefore, the current results indicate that the smallest-scale fluctuations in fully turbulence are universal, but depend on the large-scale intermittency not being fully turbulent.

Introduction

Turbulence is characterized by velocity fluctuations on a wide range of scales and frequencies. In the classical theory of turbulence, the turbulent kinetic energy transfers continuously from large to small scales, and would end at the smallest length scale of turbulence, known as the Kolmogorov dissipation scale $\eta_K \equiv (\nu^3/\langle\varepsilon\rangle)^{1/4}$. Here, ν is the kinematic viscosity of the fluid and $\langle\varepsilon\rangle$ is the mean energy dissipation rate, which equals to the average flux of energy from the energy-containing large-scale eddies down to the smallest ones in the case of statistically stationary turbulent fluid motion. However, the dissipation field $\varepsilon(\boldsymbol{x},t) = (\nu/2)(\partial_i u_j + \partial_j u_i)^2$ is driven by fluctuations of velocity gradients whose magnitudes exhibit intense spikes in both space and time, resulting in spatially intermittent regions of high turbulent dissipation within a turbulent flow field. Here, the variable u_i is the fluctuating velocity. The Kolmogorov dissipation length η_K is obtained from $\langle\varepsilon\rangle$ that does not account for the strongly intermittent nature of the dissipation rate field.

To examine the intermittency of $\varepsilon(\boldsymbol{x},t)$, Paladin and Vulpiani [1] put forward the idea of a local dissipation length scale η. A local Reynolds number $\mathrm{Re}_\eta = \eta|\delta_\eta u|/\nu$ is of order 1, where $\delta_\eta u = u(x+\eta) - u(x)$ is the longitudinal velocity increment over a separation of η. This local Reynolds number means that the inertial force $(\delta_\eta u)^2/\eta$ and the viscous force $\nu|\delta_\eta u|/\eta^2$ are local and instantaneously balanced. On the dissipation scale η all contributions from pressure, advection and the dissipation terms are assumed to be of the same order [2]. Physically, η can be interpreted as the instantaneous cut-off scale where viscosity overwhelms inertia. To capture the dynamics of the dissipation structures, the continuous distribution of dissipation scales represented by its probability density function (PDF), $Q(\eta)$, was also theoretically (e.g., [2-5]) and numerically (e.g., [6, 7]) investigated. Using the assumption that the energy flux toward small scales sets up at the integral length-scale L and the PDF of velocity increments $\delta_L u$ ($\equiv u(x+L) - u(x)$) across the integral length scale L is close to Gaussian, Yakhot derived an analytical form for $Q(\eta)$ by applying the Mellin transform to the structure function exponent relationships for moments of $\delta_\eta u$ within the range $0 < \eta < L$.

Bailey et al. [8] experimentally obtained $Q(\eta)$ using turbulent pipe flow over a wide range of Reynolds number. Their results showed reasonable agreement with theoretical predictions and with those from high resolution numerical simulations of homogeneous and isotropic box turbulence [6], which suggests a universal behavior of the smallest-scale fluctuations around the Kolmogorov dissipation scale. To test the universality of the smallest-scale fluctuations in different flows, Zhou and Xia [9, 10], and Qiu et al. [11] investigated the $Q(\eta)$ in Rayleigh–Bénard convection and Rayleigh-Taylor turbulence, respectively. Their results revealed that the distributions of η are indeed insensitive to large-scale inhomogeneity and anisotropy of the system, and confirmed that the small-scale dissipation dynamics can be described by the same models developed for homogeneous and isotropic turbulence. However, the exact functional form of $Q(\eta)$ is not universal with respect to different types of flows. Recently, Bailey et al. [12] examined the Re and mean shear dependence of $Q(\eta)$ for channel flow and found that much of the previously observed spatial dependence can be attributed to how the results are normalized.

Although the properties of $Q(\eta)$ have been investigated in several types of flows, these ideas have not been generalized for turbulent jet flows, which are widely used in various industrial mixing processes ([13-16]). In jet flows, the ambient fluid is engulfed into the main jet, resulting in "large-scale intermittency" or "external intermittency", which is related to the turbulent/non-turbulent interfaces [17, 18]. The large-scale intermittency was found to have stronger influence on the spectral inertial-range exponent than the mean shear rate. In this context, the present study investigates: (1) the properties of local dissipation scales in the centreline and in the shear layer of two jet flows, (2) the properties of large-scale velocity boundary condition in jet flows, and (3) the effect of large-scale intermittency on local dissipation scales in turbulent jets.

Description of the experiments

Experimental details for the round and square jets are given in, and the reader is directed to, Refs [13] and [19], respectively. Here a brief overview is provided. The round jet was generated from a smooth contraction nozzle with a diameter of D_e = 2 cm while the square jet issued from a square duct of dimensions 2.5 cm × 2.5 cm × 2 m, with the nominal opening area A = 6.25 cm^2 and the equivalent diameter D_e [$\equiv 2(A/\pi)^{1/2}$] ~ 2.82 cm. For the round jet, the exit velocity U_j = 3 ~ 15 m/s, which corresponds to Re ≈ 6750 ~ 20100; and for the square jet, U_j = 4.2 ~ 26.4 m/s and Re = 8×10^3 ~ 5×10^4. For both jets, the streamwise velocity was measured using single hot-wire anemometry.

The properties of small-scale turbulence were obtained using the digital filtering high-frequency noise scheme proposed by Mi et al. [20]. The dissipation and mean-square fluctuation derivatives were corrected following Hearst et al. [21]. The present hotwire probe has a limited resolution due to its finite spatial dimensions and temporal response. Specifically, its resolution was determined by the wire diameter d_w = 5 μm and effective length $\ell_w \approx$ 1 mm, plus its response frequency and sampling rate during measurements. Note that the ratio $\ell_w/d_w \approx 200$ is required so that both a nearly uniform temperature distribution in the central portion of the wire and a high sensitivity to flow velocity fluctuations can be achieved [22]. The present study corrected the spatial attenuation of the single wire due to $\ell_w \approx$ 1 mm using the procedure of Wyngaard [23], which was developed in spectral space to account for the integration effect on Fourier components of the velocity.

The present measurements consider the radial distributions of the local dissipation and PDFs of the integral length scale. These span $0 < y/y_{1/2} < 1.7$, which introduces some large scale intermittency into the signals. It has been demonstrated by Sadeghi et al.[24] that for $y/y_{1/2} > 1$, data obtained (and suitably corrected as above) using a stationary hot wire depart from those obtained in the same flow using a flying hot wire. The PDFs of local dissipation scales were calculated from each velocity time series using the following procedure, which is identical to that described in Refs [7-9].

Presentation and Discussion of Results

(1) PDFs of local dissipation scales and velocity increments along the centreline

The PDFs of local dissipation scale obtained on the jet centerline at x/D_e = 1, 5 and 30 for both the round and square jets for all the Reynolds numbers are presented in Figure 1 (a) and (b), where $Q(\eta)$ is normalized by $\eta_0 = LRe_L^{-0.72}$ [8, 9]. Here, Re_L is the Reynolds number based on the integral length scale L, i.e., $Re_L = \langle |u_x(x+L) - u_x(x)|^2\rangle^{1/2} L/\nu$.

The distributions obtained in the near and far field regions of the jet flows coincide very well with each other over all measured scales. Note that the round jet was generated from a smooth contraction nozzle while the square jet issued from a long pipe, i.e., their initial conditions are quite different. The agreement is independent of nozzle type and exit Reynolds number. This result is unexpected and surprising for many reasons.

It is well known that the vorticity layer arising from the nozzle inner wall becomes unstable, forming Kelvin–Helmholtz waves and then forming vortex rings that convect downstream. These organized vortex rings eventually break down into more complex coherent structures within a few diameters of the jet nozzle (x/D_e <5). As the flow develops downstream, the fluid entrainment becomes more stochastic in the central flow region, where incoherent small-scale turbulence plays a critical role, than in the outer region, where large-scale coherent motion dominates. As a result, the turbulence approaches near isotropy along the jet centreline in the far-filed. According to the previous studies [17, 22], both the large-scale and small-scale turbulent statistics (e.g., mean velocity decay, turbulent intensity, mean energy dissipation rate, Kolmogorov scale) in the two jets should behave somewhat differently. However, the centerline $Q(\eta)$ of jet flows appears to be independent of initial conditions. That all data agree with those from the centerline of the pipe flow of Bailey et al. [6] tends to reinforce the universality of the distribution of $Q(\eta)$.

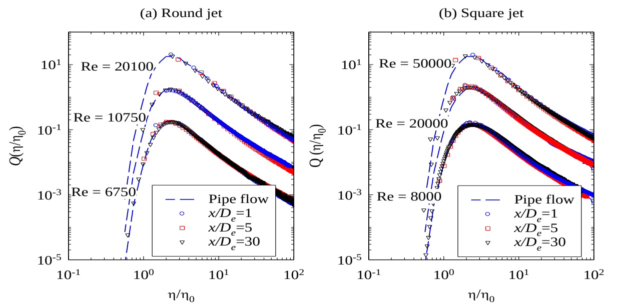

Figure 1 Centerline PDFs of the local dissipation scale obtained at x/De = 1, 3 and 30 in (a) the round jet and (b) the square jet for all the Reynolds numbers. For clarity, results for the two high Reynolds number are shifted upward by one and two decades, respectively. Results from the centreline of pipe flow [8] at Re = 24000 are also included.

In theoretical approaches [2] and numerical simulations of isotropic turbulence [6, 7], it is usually assumed that the PDF of velocity increments $\delta_L u$ ($\equiv u(x+L) - u(x)$) across the integral length scale L are Gaussian distributed, i.e., $P(\delta_L u) \sim \exp(-\delta_L u^2/2)$. Such an assumption of Gaussianity has been verified on the centreline of jet flows. Figure 2 presents $P(\delta_L u)$ measured along the centreline of the two jets. It maybe noted that the tails of the PDFs display slight lack of adherence to the Gaussian, which in the case of the round jet maybe due to the Re being on the cusp of reaching fully developed turbulence, as encapsulated in the mixing transition argument [15, 25].

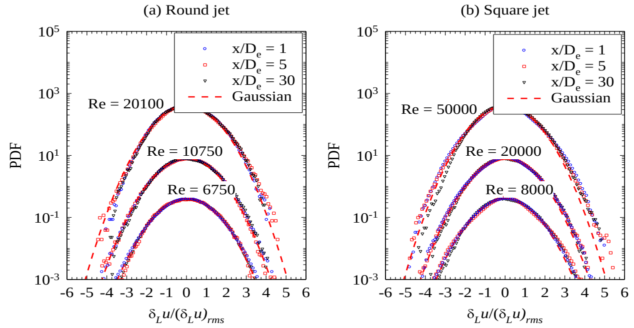

Figure 2 PDFs of $\delta_L u = u(x+L) - u(x)$ on the centreline for x/D_e = 1, 5 and 30 for (a) the round jet and (b) the square jet at different Re. Gaussian distributions are also added for reference.

(2) PDFs of local dissipation scales and velocity increments across the shear layer

Figures 3 (a) and (b) present the log-log plots of $Q(\eta)$ measured at x/D_e = 30 and at various lateral locations across the shear layer at the maximum Re considered of the two jets. The PDFs from all measurement locations collapse well for $\eta/\eta_0 \geq 3$. However, moving beyond $y/y_{1/2} \geq 0.9$, for a given value of η/η_0 indicates increased values of $Q(\eta)$. This indicates the enhanced velocity gradients at these scales and hence is a manifestation of the increased level of small-scale intermittency.

Comparison is also made in Figure 3 between the results of present jet flows, the box turbulence[6], pipe flow[8], Rayleigh – Bénard convection [9] and theoretical distribution [2]. There is a very good agreement between $Q(\eta)$ in the central “inner” layer of jet flows ($y/y_{1/2} < 0.9$) and the results of pipe flow and box turbulence. However, the $Q(\eta)$ measured at $y/y_{1/2} > 0.9$ display higher probabilities at small η with increasing $y/y_{1/2}$.

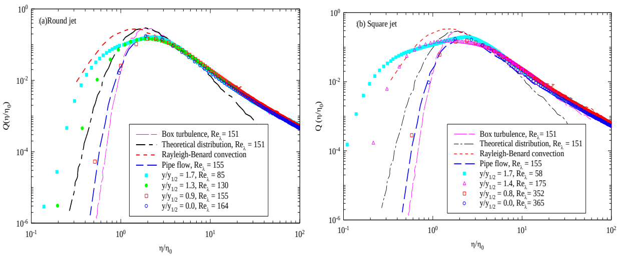

Figure 3. Measured PDFs of the local dissipation scales in the shear layer of the (a) round jet and (b) square jet for Re = 20100 and 50000, respectively. For comparison, the results from the box turbulence[6], pipe flow[8], Rayleigh – Bénard convection [9] and theoretical distribution [2] are also displayed.

Figure 4 shows P($\delta_L u$) measured at x/D_e = 30 in the shear layer of the two jets. The measured PDFs of $\delta_L u$ at $y/y_{1/2} < 0.9$ are observed to be closely Gaussian, i.e., P($\delta_L u$) ~ exp($-\delta_L u^2/2$), which was also observed by Renner et al. [26]. However, at $y/y_{1/2} > 0.9$, the PDFs of $\delta_L u$ gradually exhibit exponential tails, indicating a significant probability for the existence of much larger values than its root mean square value. Qualitatively, the measured wings can be approximated by stretched exponentials P($\delta_L u$) ~ exp($-\alpha|\delta_L u|^{\beta}$). Such exponential distribution of P($\delta_L u$) is also observed in Rayleigh–Bénard convection [9].

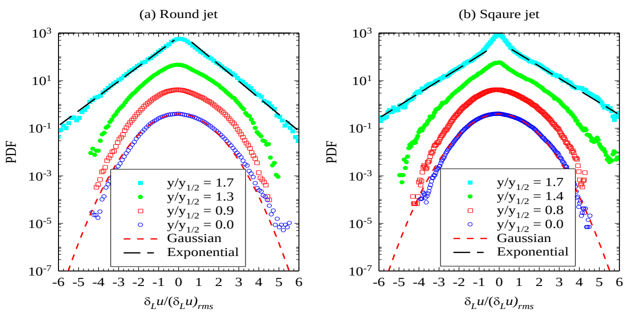

Figure 4. PDFs of $\delta_L u = u(x+L) - u(x)$ in (a) the round jet at Re = 20100 and (b) the square jet at Re = 50000. The PDFs are normalized to their respective standard deviations and shifted in the vertical direction for clarity of presentation. Gaussian and exponential distributions are shown for reference.

(3) Effect of large-scale intermittency and mean shear on PDFs of local dissipation scales and velocity increments

To understand the variation of P($\delta_L u$) and $Q(\eta)$ in the shear layer of the two jets, the large-scale intermittency factor γ (≡ the fraction of time when the flow is fully turbulent) and mean shear S ($\equiv|\partial U/\partial y|$) are considered. The turbulent energy recognition algorithm (TERA) method proposed by Falco and Gendrich [27] was applied to estimate the intermittency factor from the velocity signals of the jets. Figure 5 indicates that $\gamma \approx 1$ at $0 < y/y_{1/2} < 0.9$, the flow is fully turbulent in the jet central region. In the same region, the distributions of $Q(\eta)$ collapse with those in pipe flow [8] and box turbulence [6], see Figure 3. In addition, the P($\delta_L u$) is nearly Gaussian for $0 < y/y_{1/2} < 0.9$, see Figure 4. This means that the exact function for $Q(\eta)$ is probably universal and P($\delta_L u$) is nearly Gaussian in the fully turbulennt regions of the jets.

Figure 5 also demonstrates that the value of γ decreases quickly from 1 to 0 as $y/y_{1/2}$ increases beyond 0.9, wherein the flow is not fully turbulent as non-turbulent ambient flow is engulfed into the jet. The interfaces between the non-turbulent and turbulent regions in shear flows are investigated recently [28-30]. The most important feature of this region is the continuous exchange that occurs locally at the interface that is essential for the transport of heat, mass, and momentum between the irrotational surrounding region and the fully turbulent region of the jet. Therefore, the phenomenon of $Q(\eta)$ presenting higher probabilities at small η (Figure 4) and P($\delta_L u$) exhibiting exponential tails (Figure 5) may due to the engulfment induced large-scale intermittency.

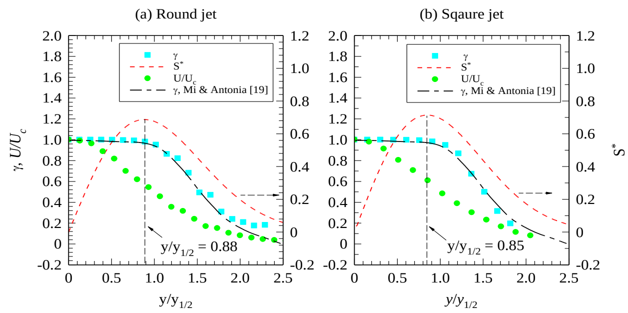

Figure 5 Radial profiles of normalized mean velocity U/U_c, the intermittency factor γ, and mean shear S^* at x/D_e = 30 of (a) the round jet and (b) square jet. The γ data of Mi and Antonia [17] are included for comparison.

To further investigate the effect of the large-scale intermittency on local dissipation scales, non-turbulent signals are identified and removed from the original velocity signals using the aforementioned TERA algorithm. Samples of the original signals are provided in Figure 6.

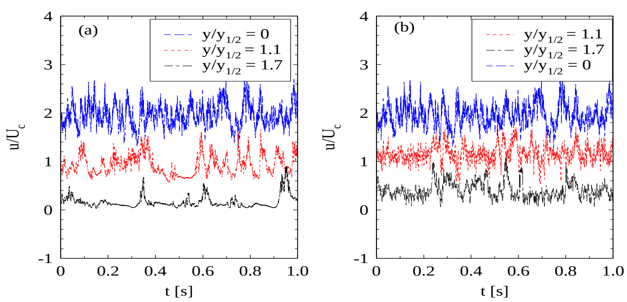

Figure 6 Plots of (a) original velocity signals, and (b) velocity signals excluding the non-turbulent parts using TERA method in the shear layer of the square jet at Re = 50000. The plots for $y/y_{1/2}$=0 and $y/y_{1/2}$=1.1 are shifted 1 and 0.5, respectively.

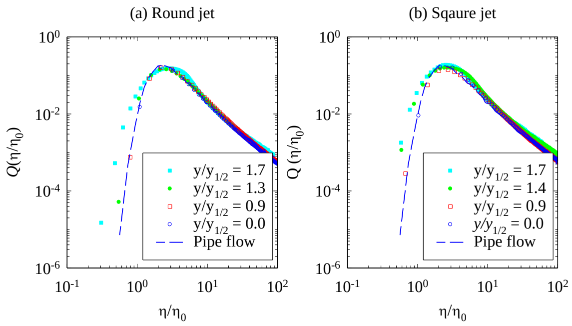

Figure 7 The PDFs of the local dissipation scales estimated from the velocity signals excluding the non-turbulent parts in the shear layer of (a) the round jet and (b) square jet for Re = 20100 and 50000, respectively. The data from pipe flow [8] are also added.

The revised PDFs of η, estimated from the velocity signals excluding the non-turbulent signals, are presented in Figure 7. In Figure 7, it is noted that the $Q(\eta)$ at small η due to the removal of the non-tubulent portion of the signal is lower than the original distributions and approach to those results obtained in the centreline of the two jet flows and pipe flow.

Conclusions

The present study has investigated the characteristics of local dissipation scale field η in turbulent round and square jets based on hot-wire measurements. From the above analysis we can induce that the non-turbulent ambient fluid engulfed into main jets causes large-scale intermittency, due to which the large-scale boundary condition, i.e., $P(\delta_L u)$, exhibits exponential tails. However, $P(\delta_L u)$ being close to Gaussian is a principle assumption used in previous theoretical and observed in the centreline of the present jet flows. Therefore, $Q(\eta)$ shows discrepancy between the shear layer and centerline of jet flows. The increased level of small-scale intermittency in the shear layer of jet flows may due to the presence of interface between the turbulence/non-turbulence regions. The excellent agreement of $Q(\eta)$ among the centerline of jet flows, pipe flow and box turbulence indicates that smallest-scale fluctuations in fully turbulence are universal, independent of turbulent intensity and isotropy.

Acknowledgements

The support of the Nature Science Foundation of China (Grants No. 51506019), and the Natural Science Foundation of Liaoning Province (Grant No. 2012025012) are gratefully acknowledged.

References

[1] Paladin, G. & A. Vulpiani, Anomalous scaling laws in multifractal objects. *Physics Reports*, **156**(4),1987,147-225.

[2] Yakhot, V., Probability densities in strong turbulence. *Physica D: Nonlinear Phenomena*, **215**(2), 2006, 166-174.

[3] Biferale, L., A note on the fluctuation of dissipative scale in turbulence. *Physics of Fluids*, **20**(3), 2008, 031703.

[4] Frisch, U. & M. Vergassola, Europhys. Lett., 1991. 14: 439.

[5] Nelkin, M., *Phys. Rev. A*, **42**, 1990, 7226.

[6] Schumacher, J., Sub-Kolmogorov-scale fluctuations in fluid turbulence. EPL (Europhysics Letters), **80**(5), 2007, 54001.

[7] Hamlington, P.E., et al., Local dissipation scales and energy dissipation-rate moments in channel flow. *Journal of Fluid Mechanics*, **701**, 2012, 419-429.

[8] Bailey, S., et al., Measurement of local dissipation scales in turbulent pipe flow. *Physical Review Letters*, **103**(1), 2009, 14502.

[9] Zhou, Q. & K.-Q. Xia, Universality of local dissipation scales in buoyancy-driven turbulence. *Physical Review Letters*, **104**(12), 2010, 124301.

[10] Xia, K. & Q. Zhou, Local Dissipation Scales and Integral-Scale Reynolds Number Scalings in Thermally-Driven Turbulence. *Journal of Physics: Conference Series*, **318**(4), 2011, 042016.

[11] Qiu, X., Y.L. Liu, & Q. Zhou, Local dissipation scales in two-dimensional Rayleigh-Taylor turbulence. *Phys Rev E Stat Nonlin Soft Matter Phys*, 90(4), 2014, 043012-043012.

[12] Bailey, S.C.C. & B.M. Witte, On the universality of local dissipation scales in turbulent channel flow. *Journal of Fluid Mechanics*, **786**, 2016, 234-252.

[13] Mi, J., M. Xu, & T. Zhou, Reynolds number influence on statistical behaviors of turbulence in a circular free jet. *Physics of Fluids*, **25**(7), 2013, 075101.

[14] Sreenivasan, K.R. & R.A. Antonia, The phenomenology of small-scale turbulence. *Annual Review of Fluid Mechanics*, **29**, 1997, 435-472.

[15] Dimotakis, P.E., Turbulent mixing. *Annual Review of Fluid Mechanics*, **37**, 2005, 329-356.

[16] Ball, C., H. Fellouah, & A. Pollard, The flow field in turbulent round free jets. *Progress in Aerospace Sciences*, 2012.

[17] Mi, J. & R.A. Antonia, Effect of large-scale intermittency and mean shear on scaling-range exponents in a turbulent jet. *Physical Review E*, **6402**(2), 2001, 026302.

[18] Zhang, J., M. Xu, & J. Mi, Large eddy simulations of a circular orifice jet with and without a cross-sectional exit plate. *Chin. Phys. B*, **23**(4), 2014.

[19] Xu, M., et al., Effects of Reynolds number on some properties of a turbulent jet from a long square pipe. *Physics of Fluids*, **25**(3), 2013, 035102.

[20] Mi, J., M. Xu, & C. Du, Digital filter for hot-wire measurements of small-scale turbulence properties. *Measurement Science and Technology*, **22**, 2011, 125401.

[21] Hearst, R., et al., Experimental estimation of fluctuating velocity and scalar gradients in turbulence. *Experiments in fluids*, **53**(4), 2012, 925-942.

[22] Mi, J. & R.A. Antonia, Vorticity characteristics of the turbulent intermediate wake. *Experiments in Fluids*, **20**(5), 1996, 383-392.

[23] Wyngaard, J., Measurement of small-scale turbulence structure with hot wires. *Journal of Physics E: Scientific Instruments*, **1**, 1968, 1105-1108.

[24] Sadeghi, H., P. Lavoie, & A. Pollard, Scale-by-scale budget equation and its self-preservation in the shear-layer of a free round jet. *International Journal of Heat and Fluid Flow*, 2016.

[25] Fellouah, H. & A. Pollard, The velocity spectra and turbulence length scale distributions in the near to intermediate regions of a round free turbulent jet. *Physics of Fluids*, **21**(11), 2009, 115101-9.

[26] Renner, C., J. Peinke, & R. Friedrich, Experimental indications for Markov properties of small-scale turbulence. *Journal of Fluid Mechanics*, **433**, 2001, 383-409.

[27] Falco, R.E. & C.P. Gendrich, The turbulence burst detection algorithm of Z. Zaric. Near-wall turbulence (A91-33726 13-34). 1990: New York, Hemisphere Publishing Corp.

[28] Khashehchi, M., et al., Evolution of the turbulent/non-turbulent interface of an axisymmetric turbulent jet. *Experiments in Fluids*, **54**(1), 2013, 1-12.

[29]Westerweel, J., et al., Mechanics of the Turbulent-Nonturbulent Interface of a Jet. *Physical Review Letters*, **95**(17), 2005, 174501.

[30] Anand, R., B. Boersma, & A. Agrawal, Detection of turbulent/non-turbulent interface for an axisymmetric turbulent jet: evaluation of known criteria and proposal of a new criterion. **47,** 2009. 995-1007.